CHAPTER-6- TRIGONOMETRY

I. 2 Marks:

1. Prove that $\sin^2 A\cos^2 B + \cos^2 A\sin^2 B + \cos^2 A\cos^2 B + \sin^2 A\sin^2 B = 1$
2. Prove that $\left[\frac{\cos^3 A-\sin^3 A}{\cos A-\sin A}\right] - \left[\frac{\cos^3 A+\sin^3 A}{\cos A+\sin A}\right] = \sin A \cos A$
3. Prove the following identities $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$
4. Two ships are sailing in the sea on either sides of a lighthouse. The angle of elevation of the top of the lighthouse as observed from the ships are 30° and 45° respectively. If the lighthouse is 200 m high, find the distance between the two ships. ($\sqrt{3} = 1.732$)
5. A road is flanked on either side by continuous rows of houses of height $4\sqrt{3}$ m with no space in between them. A pedestrian is standing on the median of the road facing a row house. The angle of elevation from the pedestrian to the top of the house is 30°. Find the width of the road.
6. The horizontal distance between two buildings is 70 m. The angle of depression of the top of the first building when seen from the top of the second building is 45°. If the height of the second building is 120 m, find the height of the first building.
7. From the top of the tower 60 m high the angles of depression of the top and bottom of a vertical lamp post are observed to be 38° and 60° respectively. Find the height of the lamp post. ($\tan 38° = 0.7813$, $\sqrt{3} = 1.732$)
8. A man is standing on the deck of a ship, which is 40 m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill. ($\sqrt{3} = 1.732$)
9. The angle of elevation of the top of a cell phone tower from the foot of a high apartment is 60° and the angle of depression of the foot of the tower from the top of the apartment is 30°. If the height of the apartment is 50 m, find the height of the cell phone tower. According to radiations control norms, the minimum height of a cell phone tower should be 120 m. State if the height of the above mentioned cell phone tower meets the radiation norms.

II. 5 Marks:

1. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$
2. If $\operatorname{cosec}\theta + \cot\theta = P$, then prove that $\cos\theta = \frac{P^2-1}{P^2+1}$
3. Prove that $\frac{\sin A}{\sec A+\tan A-1} + \frac{\cos A}{\operatorname{cosec} A+\cot A-1} = 1$
4. Prove that $\frac{(1+\cot A+\tan A)(\sin A-\cos A)}{\sec^3 A-\operatorname{cosec}^3 A} = \sin^2 A\cos^2 A$
5. If $\frac{\cos^2\theta}{\sin\theta} = p$ and $\frac{\sin^2\theta}{\cos\theta} = q$, then prove that $p^2 q^2 (p^2 + q^2 + 3) = 1$
6. Prove the following identities

   $\frac{\sin^3 A+\cos^3 A}{\sin A+\cos A} + \frac{\sin^3 A-\cos^3 A}{\sin A-\cos A} = 2$
7. If $\sqrt{3}\sin\theta - \cos\theta = 0$, then show that $\tan 3\theta = \frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}$
8. If $\sin\theta(1 + \sin^2\theta) = \cos^2\theta$, then prove that $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$

9. If $\frac{\cos\theta}{1+\sin\theta} = \frac{1}{a}$, then prove that $\frac{a^2-1}{a^2+1} = \sin\theta$

10. A TV tower stands vertically on a bank of a canal. The tower is watched from a point on the other bank directly opposite to it. The angle of elevation of the top of the tower is 58°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal. (tan 58° = 1.6003).

11. An aeroplane sets off from G on a bearing of 24° towards H, a point 250 km away. At H it changes course and heads towards J on a bearing of 55° and a distance of 180 km away.

    (i) How far is H to the North of G? (ii) How far is H to the East of G?

    (iii) How far is J to the North of H? (iv) How far is J to the East of H?

    [Sin 24° = 0.4067 sin 11° = 0.1908
    Cos 24° = 0.9135 cos 11° = 0.9816]

12. A statue 1.6 m tall stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 40°. Find the height of the pedestal.(tan 40° = 0.8391, √3 = 1.732)

13. A flag pole 'h' metres is on the top of the hemispherical dome of radius 'r' metres. A man is standing 7 m away from the dome. Seeing the top of the pole at an angle 45°, moving 5 m away from the dome, and seeing the bottom of the pole at an angle 30°. Find (i) the height of the pole (ii) radius of the dome. (√3 = 1.732)

14. The top of a 15 m high tower makes an angle of elevation of 60° with the bottom of an electronic pole and angle of elevation of 30° with the top of the pole. What is the height of the electric pole?

15. A man is watching a boat speeding away from the top of a tower. The boat makes an angle of depression of 60° with the man's eye when at a distance of 200 m from the tower. After 10 seconds, the angle of depression becomes 45°. What is the approximate speed of the boat (in km / hr), assuming that it is sailing in still water? (√3 = 1.732)

16. An aeroplane at an altitude of 1800 m finds that two boats are sailing towards it in the same direction. The angles of depression of the boats as observed from the aeroplane are 60° and 30° respectively. Find the distance between the two boats. (√ 3 = 1.732)

17. A lift in a building of height 90 feet with transparent glass walls is descending from the top of the building. At the top of the building, the angle of depression to a fountain in the garden is 60°. Two minutes later, the angle of depression reduces to 30°. If the fountain is 30 √3 feet from the entrance of the lift, find the speed of the lift, which is descending.

18. From a window (h metres high above the ground) of a house in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are θ1 and θ 2 respectively. Show that the height of the opposite house is h $h\left(1+\frac{\cot\theta_2}{\cot\theta_1}\right)$

19. If the angle of elevation of a cloud from a point 'h' metres above a lake is θ1 and the angle of depression of its reflection in the lake is θ2 . Prove that the height that the cloud is located from the ground is h$\left(\frac{\tan\theta_1+\tan\theta_2}{\tan\theta_2-\tan\theta_1}\right)$

20. The angles of elevation and depression of the top and bottom of a lamp post from the top of a 66 m high apartment are 60° and 30° respectively. Find

    (i) The height of the lamp post. (ii) The difference between height of the lamp post and the apartment.

    (iii) The distance between the lamp post and the apartment. (√3 = 1.732)