CHAPTER-1- APPLICATION OF MATRICES AND DETERMINANTS

1. If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is non-singular, find A^{-1}.

2. Decrypt the received encoded message [2 3][20 4] with the encryption matrix $\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$.

3. Show that the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & -1 \\ 5 & 2 & 1 \end{bmatrix}$ is non-singular and reduce it to the identity matrix by elementary row transformations.

4. Find the inverse of $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss-Jordan method.

5. Find the inverse of each of the following by Gauss-Jordan method: $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$.

6. If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the products *AB* and *BA* and hence solve the system of equations $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

7. The prices of three commodities *A, B, and C* are ₹ *x,y* and *z* per units respectively. A person *P* purchases 4 units of *B* and sells two units of *A* and 5 units of *C*. Person *Q* purchases 2 units of *C* and sells 3 units of *A* and one unit of *B*. Person *R* purchases one unit of *A* and sells 3 unit of *B* and one unit of *C*. In the process, *P, Q, and R* earn ` 15,000, ` 1,000 and ` 4,000 respectively. Find the prices per unit of *A, B, and C.* (Use matrix inversion method to solve the problem.).

8. In a T20 match, Chennai Super Kings needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y = ax^2 + bx + c$ with respect to a *xy* -coordinate system in the vertical plane and the ball traversed through the points (10,8),(20,16),(30,18), can you conclude that Chennai Super Kings won the match? Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is (70,0).)

9. A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadais is ₹ 150. The cost of the two dosai, two idlies and four vadais is ₹ 200. The cost of five dosai, four idlies and two vadais is ₹ 250. The family has ₹350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had?

10. The upward speed *v*(t) of a rocket at time *t* is approximated by $v(t) = at^2 + bt + c$, $0 \le t \le 100$ where *a,b and c* are constants. It has been found that the speed at times t = 3, t = 6 and t = 9 seconds are respectively,64, 133, and 208 miles per second respectively. Find the speed at time t = 15 seconds. (Use Gaussian elimination method.)

11. An amount of ₹ 65,000 is invested in three bonds at the rates of 6%, 8% and 10% per annum respectively. The total annual income is ₹ 4,800. The income from the third bond is ₹ 600 more than that from the second bond. Determine the price of each bond. (Use Gaussian elimination method.)

12. A boy is walking along the path $y = ax^2 + bx + c$ through the points (-6, 8), (-2, -12), and (3, 8). He wants to meet his friend at *P (7, 60).* Will he meet his friend? (Use Gaussian elimination method.)

13. Test for consistency of the following system of linear equations and if possible solve:

    $x + 2y - z = 3$, $3x - y + 2z = 1$, $x - 2y + 3z = 3$, $x - y + z + 1 = 0$.

14. Test for consistency of the following system of linear equations and if possible solve:

    $x - y + z = -9$, $2x - 2y + 2z = -18$, $3x - 3y + 3z + 27 = 0$.

15. Find the condition on *a*, *b* and *c* so that the following system of linear equations has one parameter family of solutions: x + y + z = a, x + 2y + 3z = b, 3x + 5y + 7z = c.
16. Investigate for what values of λ and μ the system of linear equations
    x + 2y + z = 7, x + y + $\lambda z = \mu$, $x + 3y - 5z = 5$
    has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.
17. Find the value of k for which the equations kx – 2y + z = 1, x – 2ky + z = -2, x – 2y + kz = 1 have
    (i) no solution (ii) unique solution (iii) infinitely many solutions
18. Investigate the values of λ and μ the system of linear equations 2x + 3y + 5z =9, 7x + 3y – 5z = 8, 2x + 3y + $\lambda z = \mu$ *have*
    (i) no solution (ii) a unique solution (iii) an infinite number of solutions.
19. Test for consistency and if possible, solve the following systems of equations by rank method.
    2x + 2y + z = 5, x – y + z = 1, 3x + y + 2z = 4
20. Determine the values of λ for which the following system of equations
    (3λ – 8)x + 3y + 3z = 0, 3x + (3λ -8)y + 3z = 0, 3x + 3y + (3λ – 8) = 0 has a non-trivial solution.
21. By using Gaussian elimination method, balance the chemical reaction equation:
    $C_5 H_8 O_2 \rightarrow CO_2 + H_2O$
    (The above is the reaction that is taking place in the burning of organic compound called isoprene.)
22. If the system of equations px + by + cz = 0, ax + qy + cz = 0, ax + by + rz = 0 has a non-trivial solution and p≠a, q $\neq b$, r$\neq c$, prove that $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$.
23. Solve the following system of homogenous equations.
    2x + 3y – z = 0, x – y - 2z = 0, 3x + y + 3z = 0
24. Determine the values of λ for which the following system of equations
    x + y + 3z = 0, 4x + 3y + $\lambda z = 0, 2x +$ y + 2z = 0 has
    (i) a unique solution (ii) a non-trivial solution.
25. By using Gaussian elimination method, balance the chemical reaction equation:
    $C_2 H_6 + O_2 \rightarrow H_2O + CO_2$

Theorem with Proof

26. **Theorem 1.5 (Left Cancellation Law)**

    Let *A*, *B* and C be square matrices of order *n*. If *A* is non-singular and *AB = AC*= then *B = C*.

    Proof

    Since *A* is non-singular, A^{-1}exists and $AA^{-1} = A^{-1}A = I_n$.. Taking *AB AC* = and pre-multiplying both sides by A^{-1}, we get $A^{-1}(AB) = A^{-1}(AC)$. By using the associative property of matrix multiplication and property of inverse matrix, we get *B = C*.

27. **Theorem1.6 (Right Cancellation Law)**

    Let *A*, *B and C* be square matrices of order *n*. If *A* is non-singular and *BA*= *CA*, then *B*= *C*.

    Proof

    Since *A* is non-singular, A^{-1}exists and $AA^{-1} = A^{-1} A = I_n$. Taking *BA*= *CA* and post-multiplying both sides by A^{-1}, we get $(BA) A^{-1} = (CA)A^{-1}$. By using the associative property of matrix multiplication and property of inverse matrix, we get *B*= *C*.

28. **Theorem 1.7 (Reversal Law for Inverses)**

If *A* and *B* are non-singular matrices of the same order, then the product *AB* is also non-singular and $(AB)^{-1} = B^{-1} A^{-1}$.

Proof

Assume that *A* and *B* are non-singular matrices of same order *n*. Then, $|A| \neq 0, |B| \neq 0$, both A^{-1} and B^{-1} exist and they are of order *n*. The products *AB* and $B^{-1}A^{-1}$ can be found and they are also of order *n*. Using the product rule for determinants, we get $|AB| = |A||B| \neq 0$. So, *AB* is non-singular and

$(AB)(B^{-1} A^{-1}) = (A(BB^{-1})) A^{-1} = (AI_n) A^{-1} = AA^{-1} = I_n$;

$(B^{-1} A^{-1})(AB) = (B^{-1} (A^{-1} A) B = (B^{-1} I_n) B = B^{-1} B = I_n$.

Hence $(AB)^{-1} = B^{-1} A^{-1}$.

29. **Theorem 1.8 (Law of Double Inverse)**

If *A* is non-singular, then A^{-1} is also non-singular and $(A^{-1}) = A$.

Proof

Assume that *A* is non-singular. Then $|A| \neq 0$, and A^{-1} exists.

Now $|A^{-1}| = \frac{1}{|A|} \neq 0 \Longrightarrow A^{-1}$ is also non – singular, and $AA^{-1} = A^{-1} A = I$.

Now, $AA^{-1} = I \Longrightarrow (AA^{-1}) = I \Longrightarrow (A^{-1})^{-1} \; A^{-1} = I$.

Post-multiplying by *A* on both sides of equation (1), we get $(A^{-1})^{-1} = A$.